HYPERBOLA

1. A conic section is said to be a ***hyperbola*** if it's eccentricity is greater than 1.

2. The equation of a hyperbola in the standard form is $x^2/a^2 – y^2/b^2 = 1$.

3. In the hyperbola $x^2/a^2 – y^2/b^2 = 1$, $b^2 = a^2(e^2 – 1)$.

4. For the hyperbola $x^2/a^2 – y^2/b^2 = 1$, there are two vertices A(a, 0), A′(–a, 0) ; two foci S(ae, 0), S′(–ae, 0) ; two directrices $x = \pm a/e$ and two axes of which one is transverse axis (principal axis) y = 0 and the other is conjugate axis x = 0.

5. A point (x_1, y_1) is said to be an

   i) ***external point*** to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if $\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} < 1$

   ii) ***internal point*** to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if $\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} > 1$

6. A hyperbola is said to be a ***rectangular hyperbola*** if the length of it's transverse axis is equal to the length of it's conjugate axis.

7. The eccentricity of a rectangular hyperbola is $\sqrt{2}$.

8. The hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ are called ***conjugate hyperbolas.***

9. If e_1, e_2 are the eccentricities of two conjugate hyperbolas then $e_1^2 + e_2^2 = e_1^2 e_2^2$.

10. A chord passing through a point P on the hyperbola and perpendicular to the transverse axis(Principal axis) of the hyperbola is called the ***double ordinate*** of the point P.

11. A chord of the hyperbola passing through either of the foci is called a ***focal chord.***

12. A focal chord of a hyperbola perpendicular to the transverse axis (Principal axis) of the hyperbola is called **latus rectum.** If the latus rectum meets the hyperbola in L and L′ then LL′ is called ***length of the latus rectum.***

13. The length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}$.

14. If P is a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with foci S and S′ then PS′ - PS = 2a.

15. The equation of the hyperbola whose transverse axis is parallel to x-axis and the centre at (α, β) is $\frac{(x-\alpha)^2}{a^2} - \frac{(y-\beta)^2}{b^2} = 1$.

16. The equation of the hyperbola whose transverse axis is parallel to y – axis and the centre at (α, β) is $\frac{(y-\beta)^2}{b^2} - \frac{(x-\alpha)^2}{a^2} = 1$.

17. We use the following notation in this chapter

$S \equiv \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1$, $S_1 \equiv \frac{xx_1}{a^2} - \frac{yy_1}{b^2} - 1$, $S_{11} = S(x_1, y_1) = \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1$, $S_{12} = \frac{x_1x_2}{a^2} - \frac{y_1y_2}{b^2} - 1$.

18. Let $P(x_1, y_1)$ be a point and $S \equiv \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0$ be a hyperbola . Then

i) P lies on the hyperbola $S = 0 \Leftrightarrow S_{11} = 0$

ii) P lies inside the hyperbola $S = 0 \Leftrightarrow S_{11} > 0$

iii) P lies outside the hyperbola $S = 0 \Leftrightarrow S_{11} < 0$

19. The equation of the chord joining the two points $A(x_1, y_1)$, $B(x_2, y_2)$ on the hyperbola $S = 0$ is $S_1 + S_2 = S_{12}$.

20. The equation of the tangent to the hyperbola $S = 0$ at $P(x_1, y_1)$ is $S_1 = 0$.

21. The equation of the normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $P(x_1, y_1)$ is $\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$.

22. The condition that the line $y = mx + c$ may be a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $c^2 = a^2m^2 - b^2$

23. The equation of a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ may be taken as $y = mx \pm \sqrt{a^2m^2 - b^2}$. The point of contact is $\left(\frac{-a^2m}{c}, \frac{-b^2}{c}\right)$ where $c^2 = a^2m^2 - b^2$.

24. The condition that the line $lx + my + n = 0$ may be a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $a^2l^2 - b^2m^2 = n^2$.

25. Two tangents can be drawn to a hyperbola from an external point.

26. If m_1, m_2 are the slopes of the tangents through P to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $m_1 + m_2 = \frac{2x_1y_1}{x_1^2 - a^2}$, $m_1m_2 = \frac{y_1^2 + b^2}{x_1^2 - a^2}$.

27. The locus of point of intersection of perpendicular tangents to a hyperbola is a circle concentric with the hyperbola. This circle is called ***director circle*** of the hyperbola.

28. The equation to the direction circle of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $x^2 + y^2 = a^2 – b^2$.

29. The locus of the feet of the perpendiculars drawn from the foci to any tangent to the hyperbola is a circle concentric with the hyperbola. This circle is called ***auxiliary circle*** of the hyperbola.

30. The equation to the auxiliary circle of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $x^2 + y^2 = a^2$,

31. The equation to the chord of contact of $P(x_1,y_1)$ with respect to the hyperbola S = 0 is $S_1 = 0$.

32. The equation of the polar of the point $P(x_1, y_1)$ with respect to the hyperbola S = 0 is $S_1= 0$.

33. The pole of the line lx + my + n = 0 (n ≠ 0) with respect to the hyperbola S = $\frac{x^2}{a^2}-\frac{y^2}{b^2}-1$ = 0 is $\left(\frac{-a^2 l}{n},\frac{b^2 m}{n}\right)$.

34. The condition for the points $P(x_1, y_1)$, $Q(x_2, y_2)$ to be conjugate with respect to the hyperbola S = 0 is $S_{12} = 0$.

35. Two lines $L_1 = 0$, $L_2= 0$ are said to be ***conjugate lines*** with respect to the hyperbola S = 0 if the pole of $L_1 = 0$ lies on $L_2 = 0$.

36. The condition for the lines $l_1x + m_1y + n_1 = 0$ and $l_2x + m_2y + n_2 = 0$ to be conjugate with respect to the hyperbola $x^2/a^2 – y^2/b^2 = 1$ is $a^2l_1l_2 – b^2 m_1m_2 = n_1n_2$.

37. The equation of the chord of the hyperbola S = 0 having $P(x_1, y_1)$ as it's midpoint is $S_1 = S_{11}$.

38. The equation to the pair of tangents to the hyperbola S =0 form $P(x_1, y_1)$ is $S_1^2 = S_{11}S$.

39. The tangents of a hyperbola which touch the hyperbola at infinity are called ***asymptotes*** of the hyperbola.

40. The equation of the asymptotes of the hyperbola S = 0 are $\frac{x}{a}\pm\frac{y}{b}=0$.

41. The equation to the pair of asymptotes of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$.

42. The equations to the pair of asymptotes and the hyperbola differ by a constant.
43. Asymptotes of a hyperbola passes through the centre of the hyperbola.
44. Asymptotes are equally inclined to the axes of the hyperbola.
45. The angle between the asymptotes of the hyperbola S = 0 is 2Tan^{-1} b/a or $2\sec^{-1}$ e.
46. A point (x, y) on the hyperbola $x^2/a^2 – y^2 /b^2 = 1$ represented as x = a secθ, y = btanθ in a single parameter θ. These equations x = asecθ, y = btanθ are called ***parametric equations*** of the hyperbola $x^2/a^2 – y^2/b^2 = 1$. The point (a secθ , b tanθ) is simply denoted by θ.

47. A point on the hyperbola $x^2/a^2 - y^2/b^2 = 1$ can also be represented by ($a \cosh \theta$, $b \sinh \theta$). The equations $x = a\cosh\theta$, $y = b\sinh\theta$ are also called ***parametric equation*** of the hyperbola $x^2/a^2 - y^2/b^2 = 1$.

48. The equation of the chord joining two points α and β on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{x}{a}\cos\frac{\alpha-\beta}{2} - \frac{y}{b}\sin\frac{\alpha+\beta}{2} = \cos\frac{\alpha+\beta}{2}$.

49. The equation of the tangent at $P(\theta)$ on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$.

50. The equation of the normal at $P(\theta)$ on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2$.

51. The curve represented by
$S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is
i) a pair of parallel lines if $h^2 = ab$, $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$
ii) a parabola if $h^2 = ab$, $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$
iii) an ellipse if $h^2 < ab$
iv) a circle if $a = b$, $h = 0$, $g^2 + f^2 - ac \geq 0$
v) a pair of intersecting lines if $h^2 > ab$, $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$
vi) a hyperbola if $h^2 > ab$, $abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0$.

52. The equation to the pair of asymptotes of the hyperbola $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is $ax^2 + 2hxy + by^2 + 2gx + 2fy + c - \frac{\Delta}{ab - h^2} = 0$.

53. The condition that the line $lx + my + n = 0$ to be a normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{(a^2 + b^2)^2}{n^2}$.

54. The equation of a rectangular hyperbola whose asymptotes are the coordinate axes is $xy = c^2$.

55. The parametric equation of $xy = c^2$ are $x = ct$, $y = c/t$.

56. The eccentricity of $xy = c^2$ is $\sqrt{2}$